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EFFECTIVE REFLECTING SURFACE OF AN IONIZED REGION HAVING THE SHAPE OF A SPHERE

by

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[USSR]

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EFFECTIVE REFLECTING SURFACE OF AN IONIZED REGION HAVING THE SHAPE OF A SPHERE *

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by N. P. Mar'yin

SUMMARY

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A geometrical optics method is applied in this work in order to solve the problem of plane radiowave reflection from a sphere-like plasma region, in which the dielectric constant depends only on a single coordinate **r**. The intensity of the reflected field is plotted as a function of the angle θ. The decrease in density of electromagnetic energy flux at the expense of energy absorption in the ionized region is ascertained.

AUTHOR

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INTRODUCTION

Let us consider the reflection of a plane electromagnetic wave from a sphere-like ionized region. We shall estimate, that the dimensions of the region are substantially greater than the wavelength and that the distance from the plasma sphere to the point of observation is significantly greater than the geometric dimensions of the regions. We shall place the origin of the coordinates at the center of the sphere. Then the relative dielectric constant of the ionized region, ε_r, may be represented as follows :

$$\varepsilon_r(r) = 1 - \frac{4\pi N(r) e^2}{m\omega(\omega + i\nu_{\text{эф}})}, \qquad (1)$$

* OB EFFEKTIVNOY OTRAZHAYUSHCHEY POVERKHNOSTI IONIZIROVANNOY OBLASTI, IMEYUSHCHEY FORMU SHARA.

where N (r) is the concentration of electrons; m, e are respectively the mass and the charge of the electron; ω is the cyclical frequency of the incident wave; $\nu_{эф}$ is the effective frequency of electron collisions with ions.

Since in the considered region ε_r is a function of $\underline{r}$ only, this region has a spherical symmetry. It is well known (see, for example, [1]), that in a medium with spherical symmetry, the trajectory of a ray is determined by the equation

$$\theta = \theta_n + \int_r^{r_n} \frac{\rho\, dr}{r\sqrt{r^2 n^2(r) - \rho^2}}, \tag{2}$$

where $\rho = r_n \sin\theta_n$ is the distance from the radial ray, coinciding with the direction of flat wave incidence, to the incident ray under consideration; r_n is the radius of the sphere, bounding the considered region.

If the ionized region has a blurred boundary ($\varepsilon_r \to 1$ at $r \to \infty$), then ρ is the distance from the radial ray to the still nonrefracting ray. Then the formula (2) ought to be written in the following manner :

$$\theta = \int_r^{\infty} \rho\, dr / r\sqrt{r^2 n^2(r) - \rho^2}\ . \tag{3}$$

At points r_i, where

$$r_1 n(r_1) = \rho, \tag{4}$$

the denominator of the subintegral expression becomes zero. However, the integrals (2) and (3) remain finite. The tangent to ray trajectory at these points is perpendicular to the radius; starting from these points, the rays drift away from coordinate origin.

When the value of r_1 is known, the coordinate θ_1 is determined by the equation (2) or (3).

It follows from the equality (4) that for a radial ray ($\rho = 0$), the reflection takes place at the condition $n(r_o) = 0$, or $\varepsilon(r_o) = 0$. The value of the radius ($r = r_o$), for which ε becomes zero, determines the reflecting capability of the ionized region.

1. - EFFECTIVE REFLECTING SURFACE

Assume that S_0 is the density of energy flux in incident wave; then, the energy passing through an elementary area ds_0, having a shape of a ring, situated in the plane $Z = \text{const}$ (see Fig. 1), will be

$$S_0 ds_0 = S_0 2\pi\rho d\rho.$$

The energy, reflected from the ionized region, w ill propagate between the lateral surfaces of two cones, having $2\theta_1$ and $2(\theta_1 + d\theta_1)$ for angles at apex. On the concentric sphere, described around the ionized region with a radius $r \gg r_n$, a strip of an area

$$ds = 4\pi r^2 \sin 2\theta_1 d\theta_1.$$

is cut out by the surfaces of the cones.

It follows from the law of energy preservation, that

$$S_0 ds_0 = S ds.$$

Consequently, the energy flux density at great distance from the ionized region is

$$S = S_0 \frac{\rho}{2r^2 \sin 2\theta_1 (d\theta_1/d\rho)}. \qquad (5)$$

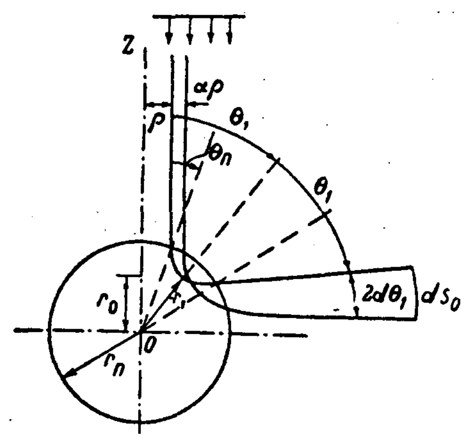

Fig. 1. - To the determination of the density of electromagnetic energy flux, reflected by the ionized region.

Therefore, contrary to the case of reflection from a metallic sphere, the density of reflected energy depends here on the direction along which the reflected rays are propagating. It is customary to compare the effective reflecting surface of any body with that of a metallic sphere, for the density of the electromagnetic energy flux, reflected from it, is independent from the angle θ, and, according to law of geometric optics [2] is

$$\underset{\sim}{S} = S_0 \frac{a^2}{4r^2} = S_0 \frac{\sigma_m}{4\pi r^2}, \qquad (6)$$

where a is the radius of the sphere.

The effective reflecting surface of a sphere is determined by the formula

$$\sigma_{\text{ш}} = \pi a^2.$$

The value of plasma region's σ is of great importance in radar location. In order to determine its order of magnitude for a sphere-like ionized region in case of radar location, when the transmitting and receiving devices are integrated at a single point, we shall investigate (5) at $\rho \to 0$ and $\theta_1 \to 0$. Taking (3) into account and evaluating the indeterminate form, we shall obtain

$$S = S_0 \frac{1}{4r^2} \frac{1}{\left[\int\limits_{r_0}^{\infty} dr/r^2 n(r)\right]^2}. \tag{7}$$

Hence, the value of the effective reflecting surface of a plasma region, on the condition that the axis of the receiving-transmitting antenna pass through the region's center and $r \gg r_0$, is

$$\sigma = \sigma_{\text{ш}} \eta, \tag{8}$$

where

$$\eta = \frac{1}{a^2 \left[\int\limits_{r_0}^{\infty} dr/r^2 n(r)\right]^2}$$

is a certain factor.

It is customary to determine the effective reflecting surface of an ionized region by way of substituting it by a metallic surface, passing wherever $\varepsilon_r = 0$, which is not always correct. For the considered region, it is a sphere of radius r_0. The expression obtained for η allows to make more precise the value of the effective reflecting surface of a sphere-like plasma region.

It follows from (8) that, provided the dielectric constant of the ionized region nowhere becomes zero, we have within the accuracy of geometric optics, $\sigma \to 0$. However, if we take into account the finite dimensions of the receiving antenna, we may obtain in it the power of the reflected wave even at $\varepsilon_r \geqslant 0$, for nonradial rays are reflected from the surface of radius $r_1 \geqslant r_0$.

Alongside with the radar case there is practical interest to determine the distribution of scattered energy in space; in the case under consideration it is determined by the expression (5).

The character of energy scattering in the plasma region depends on the law of electron distribution in it. In order to investigate in more detail the scattering of energy by the ionized region, we shall define the concrete law of variation of free elctrons' concentration in space, neglecting the losses of electromagnetic energy in the plasma.

Let us consider the cases when the density of electrons varies:

1) according to the hyperbolic law; here

$$\varepsilon_r = n^2 = 1 - (r_0^2/r^2); \tag{9}$$

2) according to the parabolic law; here

$$\varepsilon_r = n^2 = \begin{cases} 1 - b^2[1-(r^2/r_n^2)], & r \leqslant r_n; \\ 1, & r > r_n; \end{cases} \tag{10}$$

3) according to the exponential law; here

$$\varepsilon_r = n^2 = 1 - \exp[-\alpha^2(r^2 - r_0^2)]. \tag{11}$$

1.- Hyperbolic Law of N(r) Variation

Substituting (9) into (3) and integrating, we obtain

$$\theta = \frac{\rho}{\sqrt{r_0^2+\rho^2}} \arcsin \frac{\sqrt{r_0^2+\rho^2}}{r}. \tag{12}$$

Selecting r from the condition (4), we find

$$r = r_1 = \sqrt{r_0^2+\rho^2},$$

while the value of θ is, according to (12)

$$\theta_1 = (\rho/\sqrt{r_0^2+\rho^2})(\pi/2). \tag{13}$$

The dependence of energy flux density on θ_1, determined from the equation (5), taking into account (13), has the form

$$S = S_0 \frac{r_0^2}{r^2} \frac{2\theta_1}{\pi^2[1-(2\theta_1/\pi)^2]^2 \sin 2\theta_1}. \tag{14}$$

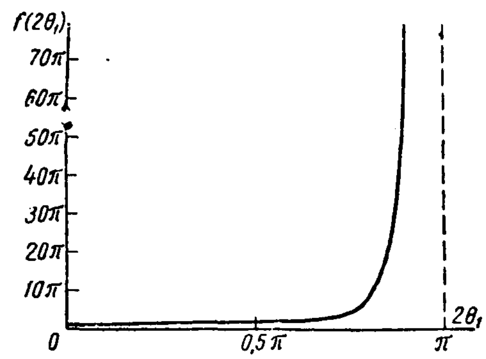

Fig. 2,- Dependence of the relative density of energy flux on the angle $2\theta_1$ for the ionization region with hyperbolic variation of the concentration of free electrons on r (the curve is symmetric relative to the axis OY)

From (14) it follows, that at reflection in opposite direction

$$S = S_0 \frac{r_0^2}{\pi^2 r^2}, \ \eta = \frac{4}{\pi^2}.$$

Scattering in directions $2\theta_1 \to \pi$ differs in that the density of energy flux increases unlimitedly. Things take place as if there were energy focusing near the shadow boundary. Plotted in Fig. 2 is the dependence of the relative density of energy flux on the angle $2\theta_1$

$$\frac{S}{S_0\,(r_0^2/\pi r^2)} = f(2\theta_1) = \frac{2\theta_1}{[1-(2\theta_1/\pi)^2]^2 \sin 2\theta_1}. \qquad (15)$$

It may be seen from the diagram, that a shadow zone forms behind the ionized region, which is considerably greater than the shadow region behind an "equivalent" sphere, having an ideally conducting surface. Therefore, the ionized region with a hyperbolic distribution of electrons along radius is endowed with a sharply expressed shadow effect.

2. - Parabolic Law of N (r) Variation

Contrary to the hyperbolic law of N (r) variation, the present case is characteristic in that ε has a finite value at the center of the region. Moreover, the ionized region with a parabolic variation of electron concentration along the radius has a limit $r = r_n$, at which the dielectric constant becomes equal to unity. The substitution of (10) into (2) gives

$$\theta_1 = \arcsin \tilde{\rho} + \frac{1}{2}\left\{\arcsin \frac{2\tilde{\rho}^2 + (b^2-1)\,\tilde{r}_1^2}{\tilde{r}_1^2 \Delta} - \arcsin \frac{2\tilde{\rho}^2 + (b^2-1)}{\Delta}\right\}, \qquad (16)$$

where

$$\tilde{r}_1^2 = \frac{r_1^2}{r_n^2} = \frac{1}{2b^2}[(b^2-1) \pm \Delta];$$

$$\tilde{\rho} = \rho/r_n; \ \Delta = \sqrt{(1-b^2)^2 + 4b^2\tilde{\rho}^2}.$$

Utilizing formula (7), we find the value of the effective reflecting surface of the plasma region, when $(1 - b^2) < 0$. It is

../..

$$\sigma = \frac{\pi r_n^2}{\left[1 + \frac{1}{(b^2-1)\,b}\right]^2}. \qquad (17)$$

At limit, when $b^2 \to \infty$, the effective reflecting surface

$$\sigma \to \pi r_n^2 .$$

Therefore, in this case the effective reflecting surface of the plasma region will be the same as that of the equivalent sphere with radius $a = r_n$.

Let us find the distribution of energy reflected in space. Since in the general case the calculations are cumbersome, we shall pause at particular examples. Assume that $b^2 = 1$. Then, according to (16), the angle $2\theta_1$ is

$$2\theta_1 = \frac{\pi}{4} + \frac{1}{2}\arcsin\tilde{\rho}, \qquad (18)$$

where

$$\tilde{\rho} = \rho/r_n.$$

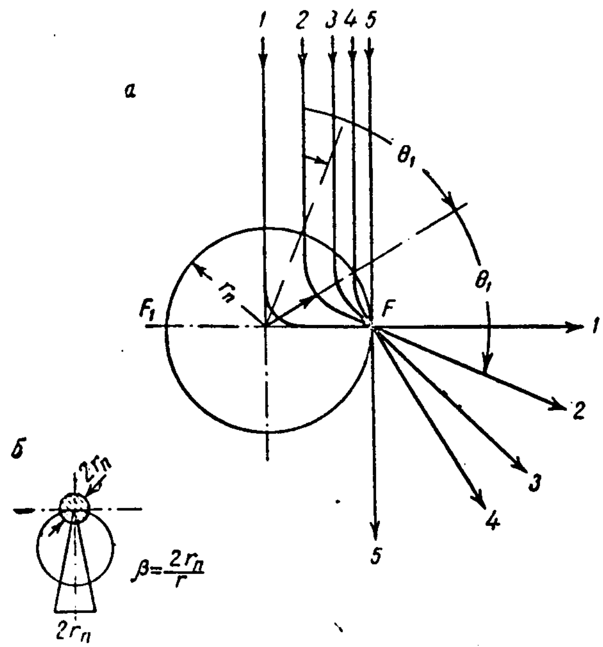

Fig. 3. - a – trajectory of rays in the ionized region, when $n = 1 - [1 - (r^2/r_n^2)]$; δ - normalized diagram of scatter.

Transforming this expression, we have

$$\sin 2\theta_1 = \sqrt{1 - \tilde{\rho}^2}.$$

It is interesting to note that all the rays, from $\tilde{\rho} = 0$ to $\tilde{\rho} = 1$ are scattered by the ionized region at angles $\pi/2 \leqslant 2\theta_1 \leqslant \pi$. In space they intersect over the circle, of which the points F and F_1 (Fig. 3a) constitute the traces in the drawing plane.

The distribution in space of incident wave's energy can be represented in the present case most appropriately as a function of

$$S = S_0 \frac{r_n^2}{r^2}\tilde{\rho}. \qquad (19)$$

This quantity S corresponds to the angle $2\theta_1$, which is determined by the formula (18); the dependence $S/[S_0(r_n^2/r^2)] = f(2\theta_1)$ for the value $b^2 = 1$ is shown in Fig. 3 δ.

It may be seen from the graph, that for great angles $2\theta_1$ the energy flux density increases. The width of the shadow lobe is

$$2\beta = 2r_n / r.$$

In the case $b^2 \gg 1$ the ionized region scatters the electromagnetic energy incident upon it uniformly into the entire space, similarly to an equivalent sphere.

3.- Ionized Region with Exponential Variation of N (r)

The scattering of electromagnetic energy by the ionized region, in which the electron density varies according to exponential law, has a great practical value. Such distribution of electrons in a cloud is most probable at natural or artificial formation of ionized media. The dielectric permeability (dielectric constant) at the center of the region is limited in magnitude; the region has no sharply defined boundary, since at $r \to \infty$, $\varepsilon_r \to 1$.

Substituting (11) into (3), we obtain

$$\theta_0 = \int_{r_1}^{\infty} \rho\, dr / r \sqrt{r^2 \{1 - \exp[-\alpha^2 (r^2 - r_0^2)]\} - \tilde{\rho}^2},$$

where r_1 is the radius, at which $r_1 \{1 - \exp[-\alpha^2 (r^2 - r_0^2)]\} = \tilde{\rho}^2$. Let us introduce a new variable $\xi = \exp[-\alpha^2 (r^2 - r_0^2)]$; then

$$\theta_1 = \int_0^{\xi_1} \alpha\rho\, d\xi / 2 (\alpha^2 r_0^2 - \ln \xi)\, \xi \sqrt{(\alpha^2 r_0^2 - \ln \xi)(1 - \xi) - \alpha^2 \rho^2}.$$

Apparently, the last integral is not taken. Let us find an approximate value for the integral, assuming that the ionized region is limited by $r = r_n$. For that case we have

$$\theta_1 \simeq \arcsin \tilde{\rho} + \int_{\xi_n}^{\xi_1} \rho_0\, d\xi / 2 (\alpha^2 r_0^2 - \ln \xi)\, \xi \sqrt{(\alpha^2 r_0^2 - \ln \xi)(1 - \xi) - \rho_0^2}, \qquad (20)$$

$$\rho_0 = \alpha\rho, \quad \xi_1 = \exp[-\alpha^2 (r_1^2 - r_0^2)], \quad \xi_n = \exp[-\alpha^2 (r_n^2 - r_0^2)].$$

In order to find the effective reflecting surface of the region in the case of radar location, when ρ_0 is small, we shall expand the

integral in (20) into Taylor series by ρ_0. As a result, we shall obtain

$$\theta_1 = \arcsin\tilde{\rho} + \rho_0 \int_{\xi_n}^{\xi_1} d\xi/2\,(\alpha^2 r_0^2 - \ln\xi)^{3/2}\,\xi\sqrt{1-\xi} +$$
$$+ \frac{\rho_0^3}{2}\int_{\xi_n}^{\xi_1} \frac{d\xi}{2\,(\alpha^2 r_0^2 - \ln\xi)^{5/2}\,\xi\sqrt{1-\xi}} + \cdots : \tag{21}$$

for small values of ρ_0 the series converges well. The series' coefficients having a form of integrals, can be computed, provided the expression $1/(\alpha^2 r_0^2 - \ln\xi)^{n/2}$ under the integral is expanded into series by $(\xi - \xi_1)$

$$\frac{1}{(\alpha^2 r_0^2 - \ln\xi)^{n/2}} = \frac{1}{(\alpha^2 r_0^2 - \ln\xi_1)^{n/2}}\left[1 + \frac{\xi - \xi_1}{2\xi_1}\,\frac{n}{\alpha^2 r_0^2 - \ln\xi_1} - \right.$$
$$\left. - \frac{(\xi-\xi_1)^2}{2\xi_1^2}\,\frac{n}{2}\,\frac{\alpha^2 r_0^2 - \ln\xi_1 - [(n+2)/2]}{(\alpha^2 r_0^2 - \ln\xi_1)^2} + \cdots\right]. \tag{22}$$

Limiting ourselves to the two first terms in (21) and the first two addends in (22), we shall obtain upon integration

$$\theta_1 = \arcsin\tilde{\rho} + \rho_0\left\{\frac{1}{2B_1^{3/2}}\left[\left(1 + \frac{3}{\xi_1 B_1}\right)(\xi_n \operatorname{arth}\sqrt{1-\xi_n} - \right.\right.$$
$$\left.\left. - \xi_1 \operatorname{arth}\sqrt{1-\xi_1}) + \frac{3}{\xi_1 B_1}\,(\sqrt{1-\xi_1} - \sqrt{1-\xi_n})\right]\right\} + \cdots,$$
$$B_1 = \alpha^2 r_0^2 - \ln\xi_1,$$

where ξ_1 is determined by the equation $1 - (\rho_0^2/B_1) = \xi_1$.

The density of energy flux, sharply reflected backward, is determined by the approximate formula at $\xi_1 = 1$

$$S = S_0\,\frac{r_0^2}{4r^2}\,\frac{1}{\left[\dfrac{r_0}{r_n} + \dfrac{1}{2}\,\dfrac{1}{\alpha^2 r_0^2}\operatorname{arth}\sqrt{1-\xi_n}\right]^2}. \tag{23}$$

Analysis of (23) shows, that the ionized region with exponential dependence of free electrons' density on r increases the density of energy flux reflected backward; as if, somehow, it focused the reflected energy. To that effect, it is necessary that

$$\alpha^2 r_0^2 \geqslant \operatorname{arth}\sqrt{1-\xi_n}/2\left(1 - \frac{r_0}{r_n}\right), \tag{24}$$

for, in opposite case, the energy would scatter.

If the coefficient a^2 increases unlimitedly, the effective reflecting surface of the ionized region approaches the value of the effective reflection surface of an equivalent sphere of radius $a = r_n$.

If the concentration of electrons is such that

$$\alpha^2 r_0^2 = \ln(4\pi N e^2 / m\omega^2) = 0,$$

the effective reflecting surface of the region is also zero.

2. - ABSORPTION OF ELECTROMAGNETIC ENERGY BY THE IONIZED REGION

The propagation of radiowaves in an ionized medium is attended by loss of wave's electromagnetic energy. The degree of density decrease of electromagnetic energy flux S_0 is characterized by the exponential multiplier

$$\exp\left[-2k\int \varkappa dl\right].$$

The density of energy flux at the current point is

$$S = S_0 \exp\left[-2k\int \varkappa\, dl\right], \qquad (25)$$

where $\varkappa$ is the absorption coefficient; dl is the rays' trajectory element; $k = 2\pi/\lambda$ is the wave number.

The integration in exponential's exponent must be effected over the entire path length, along which the ray passes in the ionized region. If at points where the ray passes the latter is endowed with such property that

$$|\varepsilon| \gg 4\pi\sigma/\omega \quad \text{and} \quad \varepsilon > 0,$$

we may approximately estimate that

$$\varkappa = 2\pi\sigma/\omega = 2\pi e^2 N(r)\nu_{э\Phi} / m\omega(\omega^2 + \nu_{э\Phi}^2)n. \qquad (26)$$

Substituting (25) into (26) and passing to the new variable $\underline{r}$, we obtain

$$S = S_0 \exp\left[-2kI(\rho, r)\right], \qquad (27)$$

where

$$I(\rho, r) = \int_r^{r_n} \varkappa(r)\, n(r)\, r\, dr / \sqrt{r^2 n^2(r) - \rho^2}.$$

Having thus determined the weakening of energy flux density, we shall find the dependence of weakening on ρ for the above-indicated models of plasma regions. At the same time, we shall choose t he integration path, equal to doubled length from the point of emission to that of ray rotation ($r = r_1$). Such choice is determined by the fact, that it is symmetrical relative to the radius passing through the rotation point. Consequently,

$$I(\rho, r) = 2\int_{r_1}^{r_n} \varkappa(r)\, n(r)\, r\, dr / \sqrt{r^2 n^2(r) - \rho^2}, \tag{28}$$

where r_1 is found from the equation $r_1^2 n^2(r_1) = \rho^2$; r_n is the radius of the sphere, limiting the region under consideration.

1. - For the hyperbolic law of variation of free electrons' concentration in the plasma region, we have

$$\varkappa(\eta) = \varkappa_0 / \eta^2 n, \tag{29}$$

$$\eta = r_0 / r; \quad n^2(\eta) = 1 - (1/\eta^2); \quad n(r_0) = 0. \tag{30}$$

Substituting (29),and (30) into (28), we shall obtain

$$I(\tilde{\rho}) = 2\varkappa_0 r_0 \pi / \sqrt{1 + \tilde{\rho}^2}, \tag{31}$$

where

$$r_0^2 = 4\pi e^2 N_0 / m(\omega^2 + \nu_{\text{эф}}^2); \quad \varkappa_0 = \nu_{\text{эф}} / 2\omega; \quad N_0 = N(r_0).$$

Therefore, the absorption of electromagnetic energy in the plasma region, or the density decrease of reflected energy flux by comparison with energy density flux obtained without taking into account the absorption, are determined by the quantity $I(\tilde{\rho})$.

2.- In the case of parabolic distribution of free electrons in the ionized region, when

$$\varkappa = \varkappa_0 b^2 \left(1 - \frac{r^2}{r_n^2}\right) \frac{1}{n(r)}, \tag{32}$$

$$\varkappa_0 = \frac{\nu_{\text{эф}}}{2\omega}, \quad n = 1 - b^2\left(1 - \frac{r^2}{r_n^2}\right), \quad b^2 = \frac{4\pi e^2 N(0)}{m(\omega^2 + \nu_{\text{эф}}^2)},$$

we shall obtain

$$I(\tilde{\rho}) = r_n b \varkappa_0 \left\{\left(1 + \frac{1 - b^2}{2b^2}\right) \ln \frac{2b\sqrt{1 - \rho^2} + 1 + b^2}{\sqrt{(b^2 - 1)^2 + 4\rho^2 b^2}} - \frac{\sqrt{1 - \rho^2}}{b}\right\}. \tag{33}$$

In case of radar location, when $\tilde{\rho} = 0$,

$$I(0) = r_n b \varkappa_0 \left\{ \left(1 + \frac{1-b^2}{2b^2}\right) \ln \frac{b+1}{|b-1|} - \frac{1}{b} \right\}. \qquad (34)$$

If $b^2 = 1$, the expression (33) has the form

$$I(\tilde{\rho}) = r_n b \varkappa_0 \left\{ \ln \frac{\sqrt{1-\tilde{\rho}^2}+1}{\tilde{\rho}} - \sqrt{1-\tilde{\rho}^2} \right\}. \qquad (35)$$

It follows from this expression, that the central rays, passing through the plasma ragion, have an anomalously high damping. If $b^2 \neq 1$, this phenomenon does not take place. As ρ rises, damping decreases; when $\tilde{\rho} = 1$, it becomes zero.

The increase in density of electrons $(b^2 > 1)$ induces the decrease in losses of electromagnetic energy for any values of $\tilde{\rho}$. When $b^2 \to \infty$, the losses decrease to zero. This is explained by the fact, that the ray hardly enters the ionized region.

3.- The exponential distribution of free electrons' concentration in the ionized region constitutes the greatest interest from the practical viewpoint, for this distribution of electron concentration corresponds to the distribution of concentration of free electrons in plasma formations encountered in the free atmosphere. In this case the absorption coefficient is

$$\varkappa(r) = \varkappa_0 \exp\left[-\alpha^2(r^2 - r_0^2)\right](1/n(r)) \qquad (36)$$

and the refractive index is

$$n(r) = 1 - \exp\left[-\alpha^2(r^2 - r_0^2)\right], \qquad (37)$$

$$I(\rho, t_1) = \varkappa_0 \int_{t_1}^{\infty} \frac{\exp\left[-\alpha^2(t - r_0^2)\right] dt}{\sqrt{t\{1 - \exp\left[-\alpha^2(t - r_0^2)\right]\} - \rho^2}} \qquad (38)$$

$r^2 = t$, where t_1 is determined from the equation

$$t_1\{1 - \exp\left[-\alpha^2(t_1 - r_0^2)\right]\} = \rho^2.$$

The integral (38) is not taken, and it can be found by numerical methods. Let us bring forth the approximate values of (38), breaking the integral into two parts :

$$I = I_1 + I_2 = \varkappa_0 \int_{t_1}^{t_n} + \varkappa_0 \int_{t_n}^{\infty}.$$

The integral I_2 can be computed approximately if we assume that $1 \gg \exp[-\alpha^2(t-r_0^2)]$; then, at $\rho^2 \geqslant r_n^2 = t_n$

$$I_2 = \varkappa_0\sqrt{\pi/\rho_0}^2 \exp[-\alpha^2(\rho^2 - r_0^2)). \quad \rho_0 = \alpha\rho.$$

For $\tilde{\rho} \sim 0$ the integral I_2 can be represented in the form

$$I_2 \simeq \varkappa_0 \int_{r_n}^{\infty} \exp[-\alpha^2(r^2 - r_0^2)]\,dr = (\varkappa_0\sqrt{\pi}/\alpha)[1-\Phi(\alpha r_n)]e^{\alpha^2 r_0^2}.$$

Taking into account the asymptotic expansion

$$1-\Phi(\alpha r_n) = \exp[-\alpha^2 r_n^2]/\sqrt{\pi}\,\alpha r_n,$$

we shall write

$$I_2 = (\varkappa_0/\alpha^2 r_n)\exp[-\alpha^2(r_n^2 - r_0^2)]. \tag{39}$$

It follows from the last expression that I_2 has a low value and is rapidly decreasing with the rise of r_n.

The integral I_1 may, by substituting $\xi = \exp[-\alpha^2(t-r_0^2)]$ be brought to the form

$$I_1 = \frac{\varkappa_0}{2\alpha}\int_{\xi_n}^{\xi_1} \frac{d\xi}{\sqrt{\alpha^2 r_0^2 - \ln\xi}\sqrt{1-\xi-[\rho^2/(\alpha^2 r_0^2 - \ln\xi)]}}.$$

Expanding $1/\sqrt{\alpha^2 r_0^2 - \ln\xi}$ and $1/(\alpha^2 r_0^2 - \ln\xi)$ into Taylor series by $(\xi - \xi_1)$, we shall obtain, respectively

$$\frac{1}{(\alpha^2 r_0^2 - \ln\xi)^{1/2}} = \frac{1}{B_1^{1/2}}\left[1 + \frac{\xi-\xi_1}{2\xi_1}\frac{1}{B_1} + \ldots\right],$$

$$\frac{1}{\alpha^2 r_0^2 - \ln\xi} = \frac{1}{B_1}\left[1 + \frac{\xi-\xi_1}{\xi}\frac{1}{B_1} + \ldots\right].$$

The series provide a good approximation when the quantity $\alpha^2 r_0^2 \gg 1$. The terms of the series decrease rapidly with the increase of term's number, not only because the quantity $(\xi - \xi_1)$ is small, but also because they are proportional to $1/B_1^n$, where $B_1 > 1$.

Keeping only the two first terms of each series, we shall represent I_1 as

$$I_1 = \frac{\varkappa_0}{2\alpha B_1^{1/2}}\int_{\xi_n}^{\xi_1}\frac{(C+D\xi)\,d\xi}{\sqrt{F-T\xi}},$$

where

$$C = 1 - \frac{1}{2B_1};\; D = \frac{1}{2\xi_1 B_1};\; F = 1 - \rho_0^2\left(\frac{1}{B_1} - \frac{1}{B_1^2}\right);\; T = 1 + \frac{\rho_0^2}{\xi_1 B_1^2}.$$

Integrating, we obtain

$$I_1 = \frac{\varkappa_0}{\alpha B_1^{1/2}} \frac{\sqrt{F - T\xi_n}}{T} \left\{ C + D \frac{(2F + T\xi_n)}{3T} \right\}.$$

For the radial ray $\rho = 0$, the weakening is determined by the quantity

$$I = I_1 + I_2,$$

where I_2 is computed with the aid of (39),

$$I = \frac{\varkappa_0}{\alpha^2 r_0} \sqrt{1 - \xi_n} \left\{ 1 - \frac{1 - \xi_n}{6\alpha^2 r_0^2} \right\} + \frac{\varkappa_0}{\alpha^2 r_n} e^{-\alpha^2 (r_n^2 - r_0^2)}. \tag{40}$$

It follows from formula (40) that the increase of αr_0 induces the decrease in losses. It should, however, be borne in mind, that losses decrease substantially at the expense of the increase of mainly α, for the ray's path length in the ionized region from the limit r_n to r_1 then decreases. Besides, it may be seen from (40), that $I_1 \gg I_2$, and the losses are determined by I_1.

***** THE END *****

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